Control systems

Lecture-3 : Time domain analysis

V. Sankaranarayanan

Outline

1. Review of Laplace Transform
2. Time domain analysis
   - First order system
   - Second order system
   - Steady state errors
3. Assignment

Review of Complex Function

Complex Variable

A complex variable has both real and imaginary part variable. It is generally represented as

$$s = \sigma + j\omega$$

Complex Function

A complex function $G(s)$ is a function of complex variable s which can be represented as

$$G(s) = G_x + jG_y$$

where G_x and G_y is the real and imaginary component respectively.

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Impulse Function

Impulse function is a special limiting class of pulse function. It can be defined as

$$
\begin{aligned}
g(t) &= \lim_{a \to 0} \frac{A}{a} && \text{for } 0 < t < a \\
&= 0 && \text{for } t < 0 \text{ and } t > a
\end{aligned}
$$

The height of function is A/a and duration is a. Area under the function is A.

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The height of function is A/a and duration is a. Area under the function is A. Laplace Transform of impulse function is

$$\begin{aligned} \mathcal{L}[g(t)] &= \lim_{a\to 0}[\frac{A}{as}(1-e^{-as})] \\ &= \lim_{a\to 0} \frac{\frac{d}{da}A(1-e^{-as})}{\frac{d}{da}(as)} \\ &= \frac{As}{s} \\ &= A \end{aligned}$$

Common Laplace Transform Pairs

SL. No.	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $u(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$t^n \ (n = 1, 2, 3..$	$\frac{n!}{s^{n+1}}$
5	e^{-at}	$\frac{1}{s+a}$
6	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$

Laplace Transform Theorems

SL. No.	Theorem	Remark
1	$\mathcal{L}[k_1 f_1(t) + k_2 f_2(t)] = k_1 F_1(s) + k_2 F_2(s)$	Linearity
2	$\mathcal{L}[e^{-at} f(t)] = F(s+a)$	Frequency Shift theorem
3	$\mathcal{L}[f(t-T)] = e^{-sT} F(s)$	Time Shift theorem
4	$\mathcal{L}[\frac{d}{dt} f(t)] = sF(s) - f(0)$	Differentiation theorem
5	$\mathcal{L}[\int_0^t f(t)dt] = \frac{F(s)}{s}$	Integration theorem
6	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$	
7	$\mathcal{L}[f(\frac{t}{a})] = aF(sa)$	Scaling theorem

Final Value Theorem

Defination

If $f(t)$ and $df(t)/dt$ are Laplace transformable and if $\lim_{t\to\infty} f(t)$ exist then

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Initial Value Theorem

Defination

If $f(t)$ and $df(t)/dt$ are Laplace transformable and if $\lim_{s\to\infty} sF(s)$ exist then

$$f(0+) = \lim_{s\to\infty} sF(s)$$

Transfer function

Representation

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}$$

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- $m = n$ - Proper transfer function

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Convolution integral

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$G(s) = \frac{Y(s)}{U(s)}$

$$y(t) = \int_0^t g(t-\tau)u(\tau)d\tau$$

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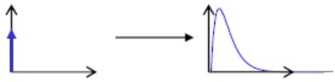

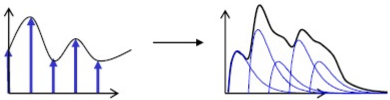

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System is said to be time invariant if

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Definition

System is said to be CAUSAL if $\tau \leq t$

Poles and Zeros

Standard form

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}$$

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Poles

Solution to the denominator polynomial called "poles"

$$a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0$$

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$$b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m = 0$$

Pole-Zero format

$$G(s) \frac{(s+z_1)(s+z_2) + \ldots + (s+z_m)}{(s+p_1)(s+p_2) + \ldots + (s+p_n)}$$

Type and order of the system

Definition

The highest power of the denominator polynomial is defined as order of the system

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Example

First order system

$$G(s) = \frac{1}{s+1}$$

Second order system

$$G(s) = \frac{s}{s^2 + 2s + 4}$$

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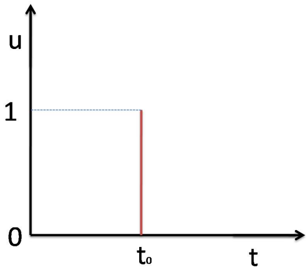
Example

Type 1 system

$$G(s) = \frac{1}{s(s+2)}$$

Impulse



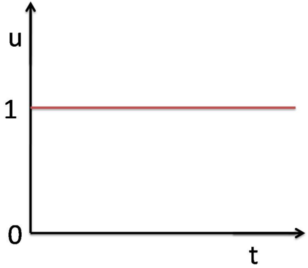
$$u(t) = \lim_{t_0 \to 0} \frac{1}{t_0} \qquad \text{for } 0 < t < t_0$$

$$= 0 \qquad \text{for } t < 0 \text{ and } t > t_0$$

$$\int_0^{\infty} u(t)dt = 1$$

$$\boxed{U(s) = 1}$$

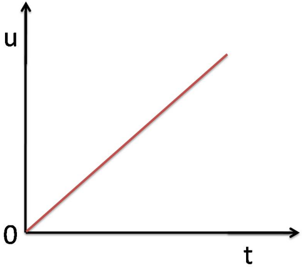
Step



$$u(t) = 1$$

$$\boxed{U(s) = \frac{1}{s}}$$

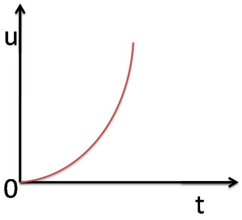
Ramp



$$u(t) = t$$

$$\boxed{U(s) = \frac{1}{s^2}}$$

Parabolic



$$u(t) = t^2$$

$$\boxed{U(s) = \frac{1}{s^3}}$$

First order system - step input

First order system

$$G(s) = \frac{1}{Ts + 1}$$

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The output response for a step input

$$Y(s) = U(s) * G(s)$$

$$Y(s) = \frac{1}{s} * \frac{1}{Ts+1}$$

$$Y(s) = \frac{1}{s} - \frac{T}{Ts+1}$$

Taking inverse Laplace transform

$$y(t) = 1 - e^{-t/T}$$

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at $t = T$

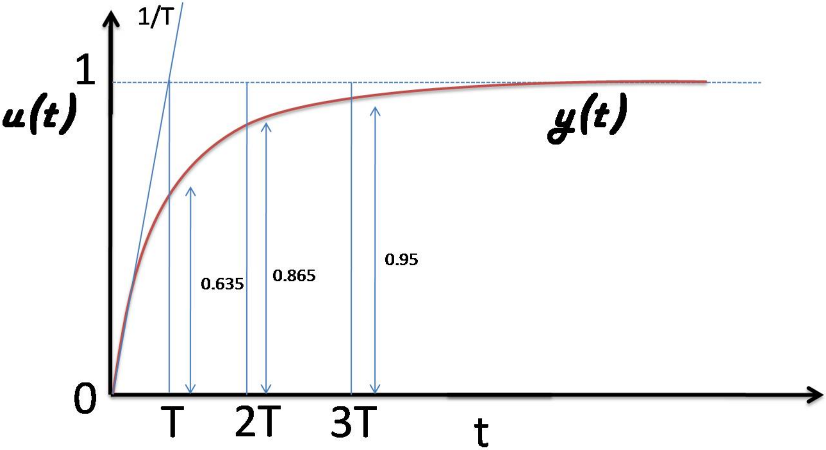
$$y(T) = 1 - e^{-1} = 0.632$$

at $t = 2T$

$$y(2T) = 0.865$$

$$\frac{dy}{dt}\Big|_{t=0} = \frac{1}{T}e^{-t/T} = \frac{1}{T}$$

First order system response - step input



First order system - ramp input

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$$y(t) = t - T + Te^{-t/T}$$

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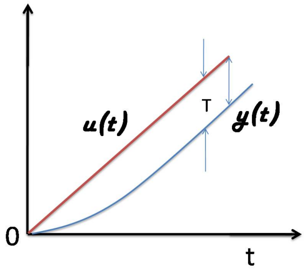

First order system - impulse input

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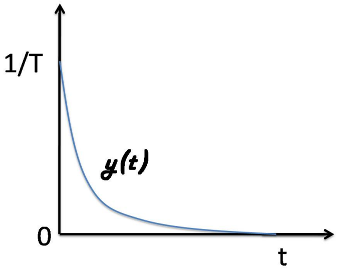


Second order system

Standard representation

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

- ω_n - Natural frequency
- ζ - Damping factor

Pole-zero form

$$\frac{\omega_n^2}{(s + \zeta\omega_n + j\omega_d)(s + \zeta\omega_n - j\omega_d)}$$

- $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ - Damped natural frequency

Second order system - step response

- $0 < \zeta < 1$
- Step response

$$Y(s) = \frac{\omega_n^2}{(s+\zeta\omega_n+j\omega_d)(s+\zeta\omega_n-j\omega_d)s}$$

- Partial fraction

$$\frac{1}{s} - \frac{s+\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2} - \frac{\zeta\omega_n}{(s+\zeta\omega_n)^2+\omega_d^2}$$

- Taking inverse Laplace transform

$$y(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_d t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_d t\right)$$

- further

$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Second order system - step response

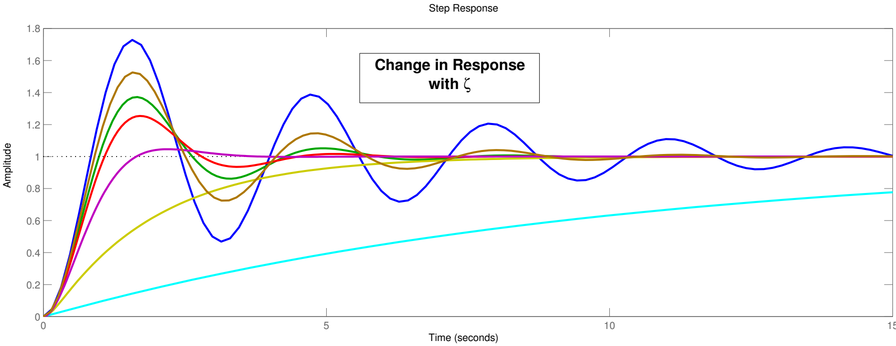
- $\zeta = 1$
- Step response

$$Y(s) = \frac{\omega_n^2}{(s+\omega_n)^2 s}$$

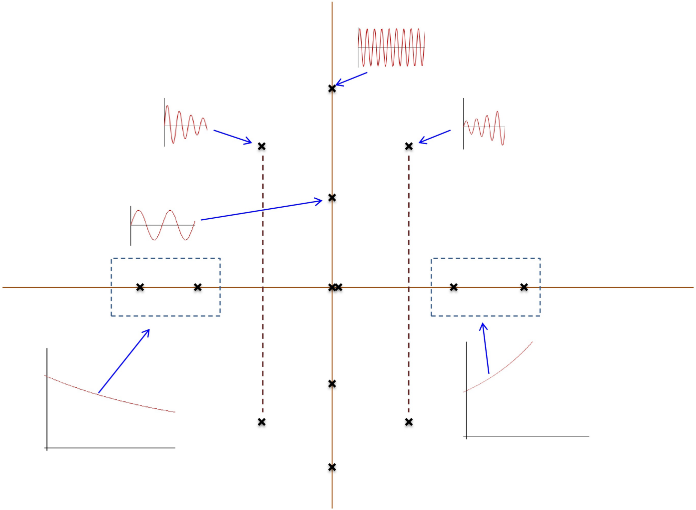
- Taking inverse Laplace transform

$$y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$

Response


Step Response
Change in Response with ζ
Amplitude
Time (seconds)
0
5
10
15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

System Resopnse With Pole Position



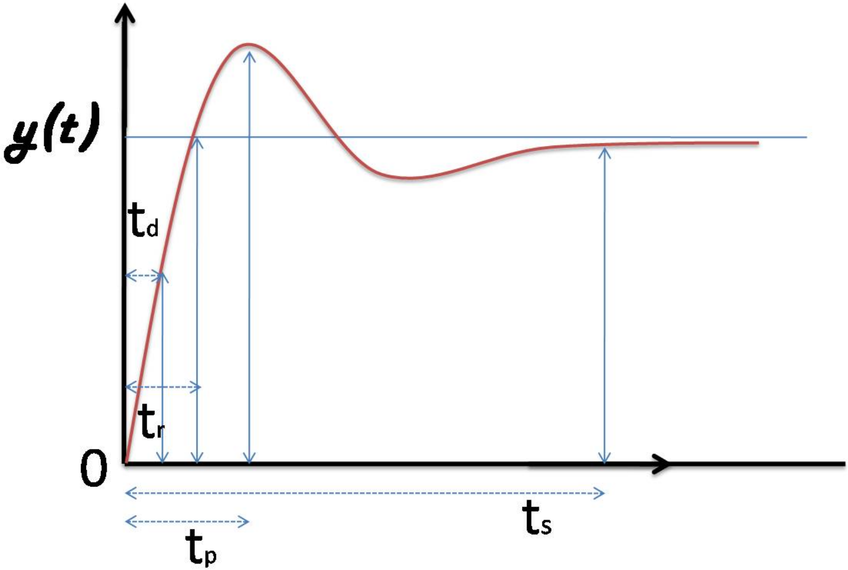
BASIC DEFINITIONS

- Delay time t_d - The delay time is the time required for the response to reach half of the final value the very first time
- Rise time t_s - The rise time is the time required for the response to rise from $0 - 100\%$ of its final value
- Peak time t_p - The peak time is the time required for the responce to reach the first peak of the overshot
- Maximum overshoot M_p

$$\frac{y(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

- Settling time t_s - Settling time is the time required for the response curve to reach and stay with in the range about the final value

Second order step responce



Time domain specifications

Rise time

$$t_r = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\omega_d}{-\zeta\omega_n}\right)$$

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Peak time

$$t_p = \frac{\pi}{\omega_d}$$

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Maximum overshoot

$$M_p = e^{-\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\pi} \times 100\%$$

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Peak time

$$t_p = \frac{\pi}{\omega_d}$$

Maximum overshoot

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Settling time

$$t_s = \frac{4}{\zeta\omega_n}$$

System With Multiple Poles and Zeros

Consider a generalized transfer function

$$G(s) = k\frac{\sum_{i=1}^{i=m}(s - z_i)}{\sum_{i=1}^{i=n}(s - p_i)}$$
$$= \frac{c_1}{s - p_1} + \frac{c_2}{s - p_2} + ... + \frac{c_n}{s - p_n} \text{if all poles are distinct}$$

where $c_1, c_2...c_n$ are the residue of $G(s)$ at p_i. So the inclusion of zeros only afftecting the values of c_i.
For poles with multiplicity, partial fraction can be done in similar way with some change in computation of c.

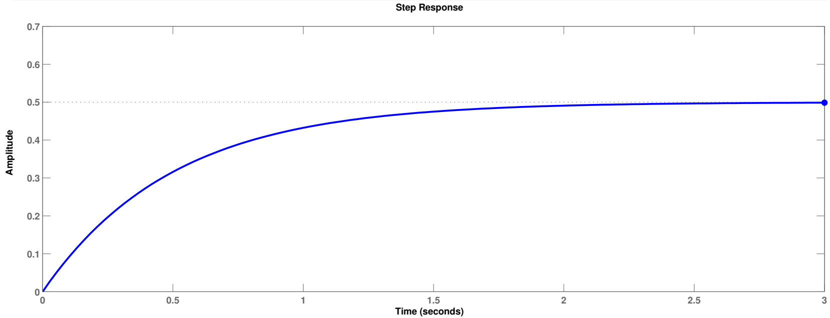
System Response with Zeros

Consider the step response of the following systems

No Zero

$$G(s) = \frac{1}{s+2} \qquad U(s) = \frac{1}{s}$$

$$y(t) = \frac{1}{2}(1 - e^{-2t})$$



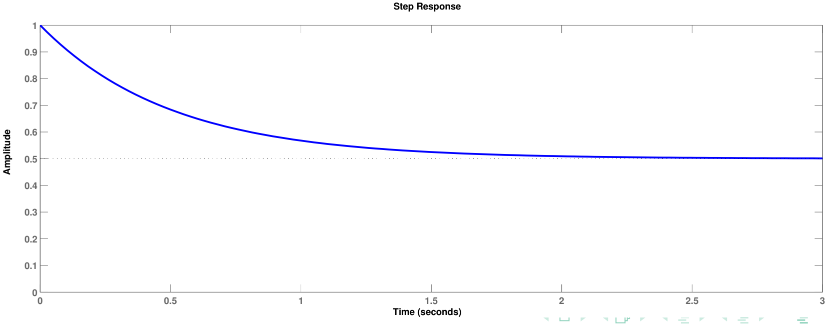
With Zero

$$G(s) = \frac{s+1}{s+2} \qquad U(s) = \frac{1}{s}$$

$$Y(s) = \frac{s+1}{s(s+2)}$$

$$= \frac{0.5}{s} + \frac{0.5}{s+2}$$

$$y(t) = 0.5 + 0.5e^{-2t}$$



System Response with Zeros

Response with Zero

Let $Y(s)$ be the response of a system $G(s)$, with unity in the numerator. The response of $(s+a)G(s)$ is

$$(s+a)Y(s) = sY(s) + aY(s)$$

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- First part is the derivative of the original response.
- Second part is a scaled version of the original response.

System Response with Zeros

Response with Zero

Let $Y(s)$ be the response of a system $G(s)$, with unity in the numerator. The response of $(s+a)G(s)$ is

$$(s+a)Y(s) = sY(s) + aY(s)$$

- First part is the derivative of the original response.
- Second part is a scaled version of the original response.
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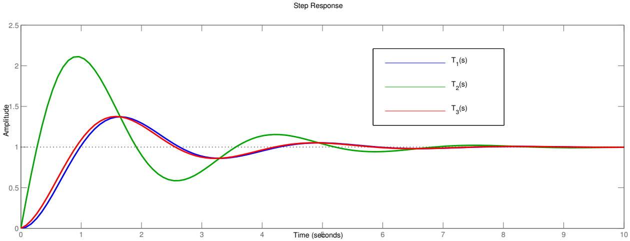
System Response with zeros

Example

Consider the following systems

$$T_1(s) = \frac{4}{s^2 + 1.2s + 4} \qquad T_2(s) = \frac{4(s+1)}{s^2 + 1.2s + 4} \qquad T_3(s) = \frac{4(s+15)}{15(s^2 + 1.2s + 4)}$$

The response of systems are



$$y_1(t) = 1 - 0.95393e^{-0.6t}\sin\left(1.9078t + 72.54^o\right)$$

$$y_2(t) = 1 - e^{-0.6t}(\cos 1.9078t - 1.7820\sin 1.9078t)$$

$$y_2(t) = 1 - e^{-0.6t}(\cos 1.9078t + 0.1747\sin 1.9078t)$$

Concept of Dominant Poles

Consider a system

$$G(s) = \frac{1}{(s+a)(s^2+bs+c)}$$

Impulse response of the system is

$$y(t) = Ae^{-at} + \quad \text{2nd order response}$$

- If a is very far from imaginary axis the exponential responce will die very fast.

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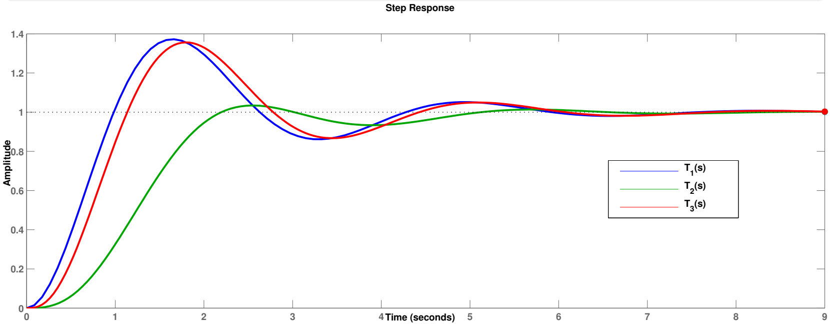
$$\text{Poles} = -0.6 \pm 1.9078i \qquad \text{Poles} = -1, -0.6 \pm 1.9078i \qquad \text{Poles} = -7, -0.6 \pm 1.9078$$



$$y_1(t) = 1 - 0.95393e^{-0.6t}\sin\left(1.9078t + 72.54^o\right)$$

$$y_2(t) = 1 - 1.0526e^{-t} - e^{-0.6t}(0.5351\sin 1.9078t - 0.0526\cos 1.9078)$$

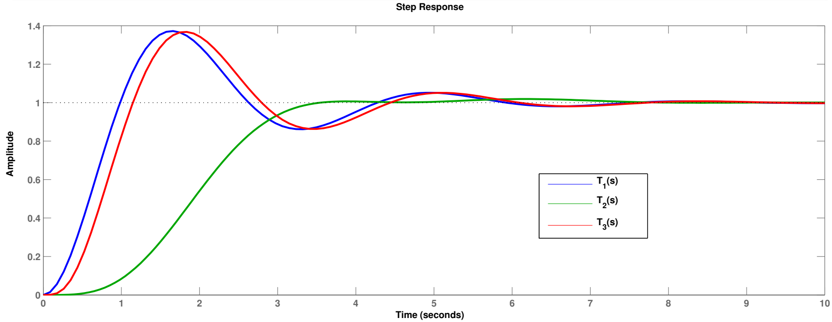
$$y_3(t) = 1 - 0.085e^{-7t} - e^{-0.6t}(0.9103\cos 1.9078t + 0.6153\sin 1.9078t)$$

Example

Consider the following systems

$$T_1(s) = \frac{4}{s^2+1.2s+4} \qquad T_2(s) = \frac{4}{(s^2+1.6s+1)(s^2+1.2s+4)} \qquad T_3(s) = \frac{400}{(s^2+16s+100)(s^2+1.2s+4)}$$

Poles $= -0.6 \pm 1.9078i$ Poles $= -0.8 \pm 0.6, -0.6 \pm 1.9078i$ Poles $= -8 \pm 6, -0.6 \pm 1.9078$

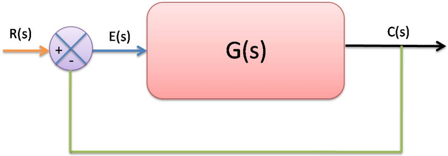


$$y_1(t) = 1 - 0.95393e^{-0.6t}\sin(1.9078t + 72.54^o)$$
$$y_2(t) = 1 - e^{-0.8t}(1.51\sin 0.6t + 1.314\cos 0.6t) + e^{-0.6t}(0.023\sin 1.9078t + 0.314\cos 1.9078t)$$
$$y_3(t) = 1 + e^{-8t}(0.028\sin 6t - 0.067\cos 6t) - e^{-0.6t}(0.9328\cos 1.9078t + 0.6633\sin 1.9078t)$$

Open-loop

$$G(s) = \frac{Y(s)}{U(s)}$$

Closed-loop system



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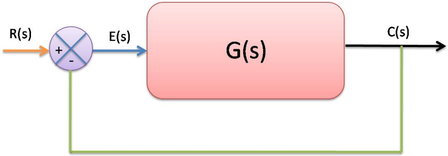
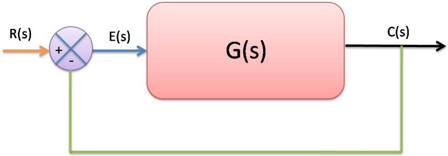


$$G(s) = \frac{Y(s)}{U(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

Closed-loop system

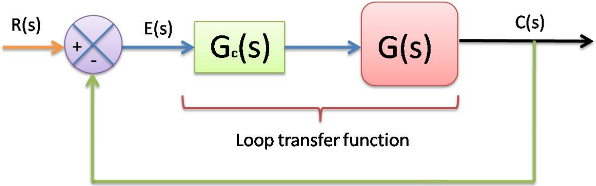


$$G(s) = \frac{Y(s)}{U(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)}$$

Closed-loop with controllers



Error

- The main of any closed loop control system is to make the error between the desired output and the actual output is zero

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- Steady state error

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

Type 0 system

Type 0 system

$$G(s) = \frac{K(T_{z1}s+1)(T_{z2}s+1)+\ldots(T_{zm}s+1)}{(T_{p1}s+1)(T_{p2}s+1)+\ldots+(T_{pn}s+1)}$$

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Parabolic input

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Type 2 system

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Steady state error constants

Type	Step Input	Ramp input	Parabolic input
Type 0 system	$\frac{1}{1+K}$	∞	∞
Type 1 system	0	$\frac{1}{K}$	∞
Type 2 system	0	0	$\frac{1}{K}$

Assignment

1. Show that $\mathcal{L}[-tf(t)] = \dfrac{dF(s)}{d(s)}$, where $F(s) = \mathcal{L}[f(t)]$

Assignment

2. Apply final value theorem to the system transfer function

$$G(s) = \frac{160s^3 + 188s^2 + 29s + 1}{260s^3 + 268s^2 + 46s + 2}$$

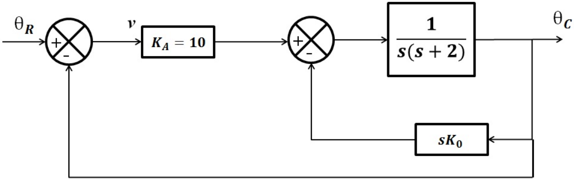
Assignment

3. Find the steady state error of the unity feedback system shown below to a step and parabolic inputs.

$$G(s) = \frac{10}{s^2(s+1)}$$

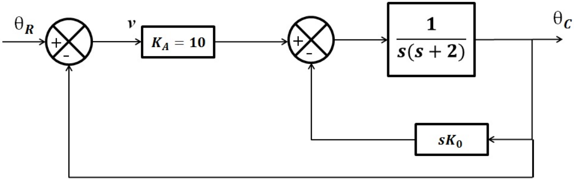
Assignment

4. Consider the system shown in the figure. If $K_0 = 0$, determine the damping factor and natural frequency of the system. What is the steady-state error resulting from unit ramp input?



Assignment

5. Consider the system shown in the figure. Determine the feedback constant K_0, which will increase damping factor of the system to 0.6. What is the steady-state error resulting from unit ramp input with this setting of the feedback constant.



Assignment

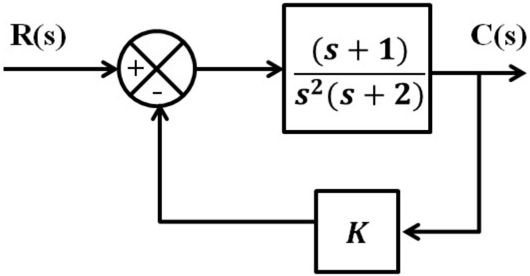
6. A certain system is described by the differential equation

$$\ddot{y} + b\dot{y} + 4 = r$$

Determine the value of b such that M_p to be as small as possible but not greater than 15%.

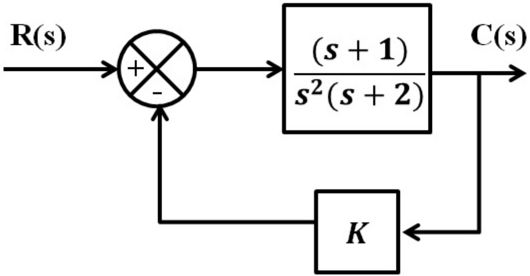
Assignment

7. Consider the system shown in the figure. Determine the system type.



Assignment

8. Consider the system shown in the figure .To yeild 0.1% error in the steady state to step input find the value of K.



Assignment

9. A unity feedback system is characterized by the open loop transfer function.

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

Determine the steady state state errors for the unit step, unit ramp and unit acceleration inputs. Also determine the damping ratio and natural frequency of the dominant roots.

Assignment

10. A speed control system of an armature-controlled dc motor as shown in the figure . uses the back emf voltage of the motor as a feedback signal. (i) Calculate the steady-state error of this system to a step input command setting the speed to a new level. Assume that R_a=L_a=J=b=1, the motor constant is K_m=1, and K_b=1. (ii) Select a feedback gain for the back emf signal to yield a step response with an overshoot of 15%.