Control systems

Frequency domain analysis: Nyquist Stability Criterion

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Outline

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1. Introduction
2. Stability Criterion

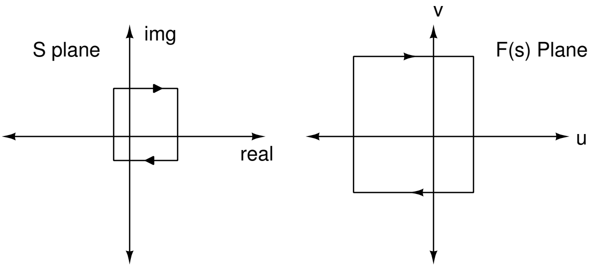
Mapping Countours in S-Plane

Mapping

A contour map is a contour or trajectory in one plane mapped or translated into another plane by a relation $F(s)$.

$$s = \sigma + j\omega$$

$$F(s) = u + jv$$



Cauchy's Theorem

Statement

If a contour in the s plane encircle Z zeros and P poles of $F(s)$ and don't pass through any poles and zeros of $F(s)$ and traverse in the clockwise direction along the contour, the corresponding contour in $F(s)$ plane encircle the origin of $F(s)$-plane $N = Z - P$ times in clockwise direction.

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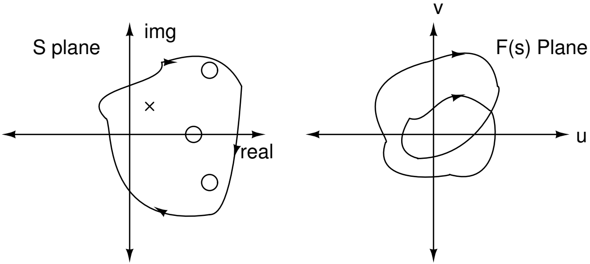

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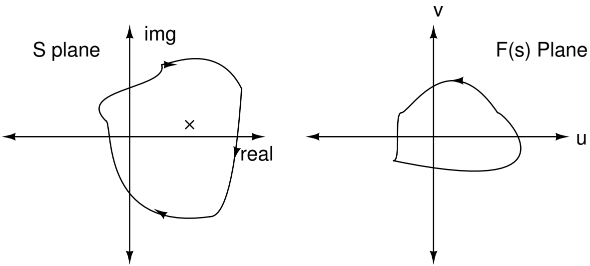

INTRODUCTOIN

Suppose

$$F(s) = 1 + G(s)$$

where G(s) is loop transfer function of a feedback system.

$$G(s) = \frac{N(s)}{D(s)}$$

$$F(s) = 1 + \frac{N(s)}{D(s)} = \frac{D(s) + N(s)}{D(s)}$$

Poles of $F(s)$ is same as poles of $G(s)$.
The zeros of the F(s) is the roots of the characteristic equation that determine the stability of the system.
Objective: To find out if any zeros of $F(s)$ lies on the right half of s-plane.

Criterion

Note that

$$F(s) = 1 + G(s)$$

and we have nyquist plot(mapping of nyquist contour) to G(s) plane and we need to find the zeros of F(s).

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So the number of encirclment of mapping of G(s) to (-1,0) in G(s) plane is the number of encirclment of mapping of F(s) to origin in the F(s) plane.

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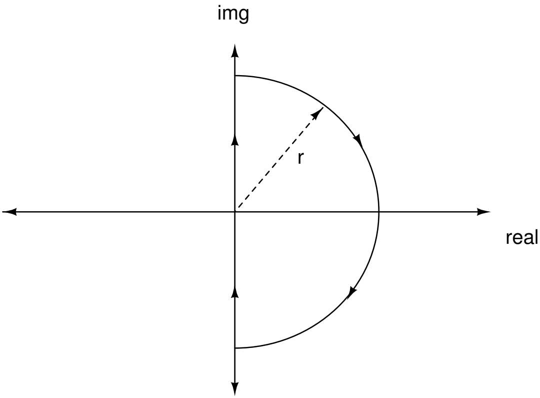
Let the number of encirclment of G(s) to (-1,0) is N.
Poles of G(s)=Poles of F(s)= P
Zeros of F(s) =Z (Unknown)

$$N = Z - P \quad Z = N + P$$

Nyquist Contour

As we need the numbers of Zeros in right half plane, We define nyquist contour in s-plane as given in the figure below. It cover entire right half so that we can chech presense of any zeros of $F(s)$ in right half leading to unstability.



$r = \infty$

Note: It consist of polar plot in positive frequency and negative frequency and a ∞ radius circle.

Criterion

Stability Condition

If $Z = 0$ the the system is stable.

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When P=0

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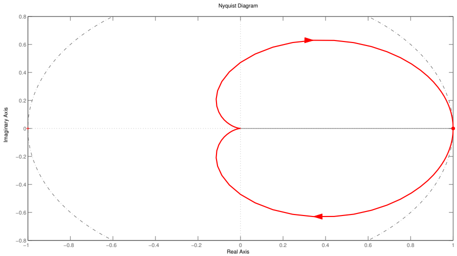
When $P \neq 0$

A feedback system is stable if and only if the mapping of nyquist contour of $G(s)$ encircle the (-1,0) point in anti clockwise direction equals to the number of pole of $G(s)$.

Example

$$G(s) = \frac{7.5}{(s+5)(s+2.5)}$$

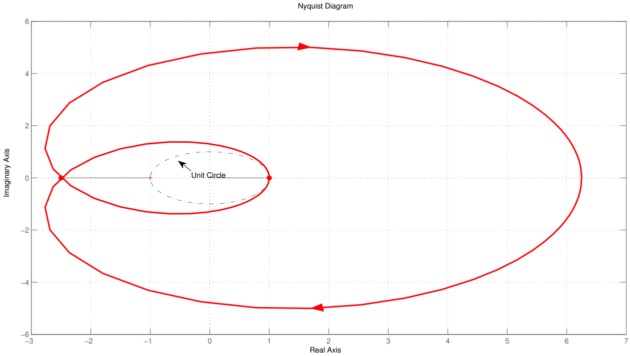
The $j\omega$ axis is mapped as in polar plot. And $-j\omega$ axis is the mirror image of the $j\omega$ axis plot. The semi-circle with $r = \infty$ maps to origin.



Close Loop system is stable

Example

$$G(s) = \frac{(s-10)(s-15)}{(s+4)(s+6)}$$



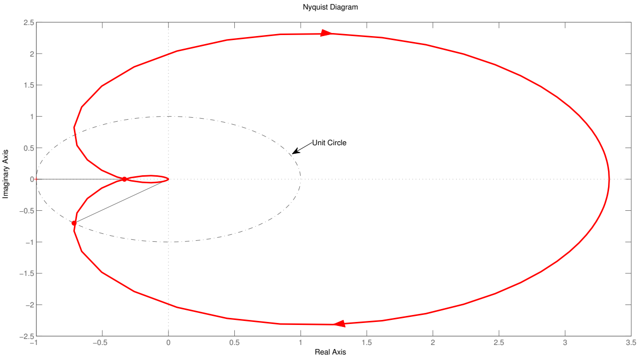
$$N = 2$$

$$Z = 2$$

System is Unstable

Example

$$G(s) = \frac{20}{(s+1)(s+2)(s+3)}$$



$$N = 0$$

$$Z = 0$$

System is stable

System with pole at origin

$$G(s) = \frac{1}{s(0.4s + 1)}$$

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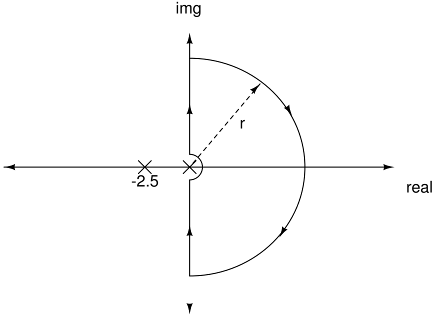
Nyquist contour must not pass through any pole and zero of $G(s)$. So we have taken a detour around origin with radius $\epsilon \to \infty$.



System with pole at origin

$$G(s) = \frac{1}{s(0.4s+1)}$$

Origin of s-plane:

$$s = \epsilon e^{j\phi}, \phi varies from - 90^o to 90^o and \epsilon \rightarrow \infty$$

As s approaches to zero G(s) is

$$lim_{\epsilon \rightarrow 0} G(S) = lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} e^{-j\phi}$$

Therefore radius of map is ∞ and angle of the map changes from 90^o at $\omega = 0_-$ to -90^o at $\omega = 0_+$. A circle with infinity radius in clockwise direction.

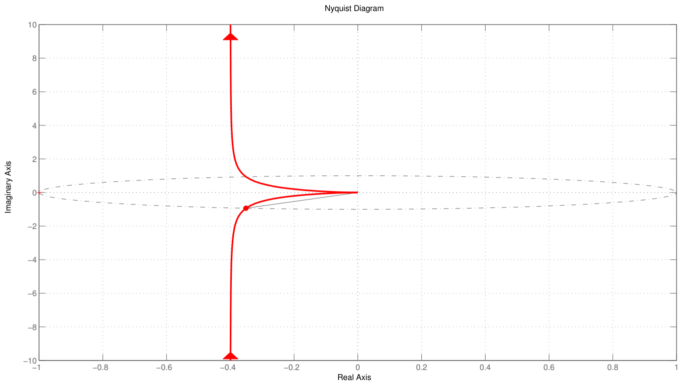
System with pole at origin

$$G(s) = \frac{1}{s(0.4s+1)}$$

From $\omega = 0_+$ to ∞ it is normal polar plot and for negative frequency it is mirror image. The semi circle with radius ∞ maps to the origin.

System with pole at origin

$$G(s) = \frac{1}{s(0.4s+1)}$$



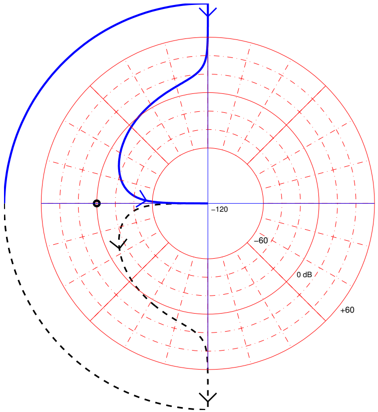
Circle with infinity radius is not shown.

Closed loop system is stable

Example

Conside

$$G(s) = \frac{1}{s(s-1)}$$



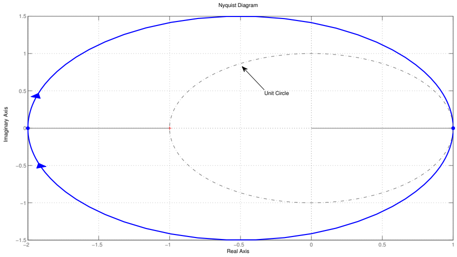
$$P = 1 \quad N = 1$$

$$Z = 2$$

Close loop system is unstable.

Example

$$G(s) = \frac{s-2}{(s+1)^2}$$



$$P = 0 \quad N = 1$$

$$Z = 1$$

Close loop system is unstable.