Modern Control systems

Lecture-1 Introduction

V. Sankaranarayanan

Outline

1. State space
   - System Description
   - Linear System
   - Introduction to State Space

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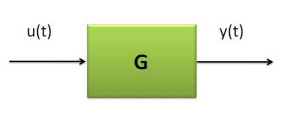
System description

Input output form



$$G : \mathbb{R}^m \to \mathbb{R}^p$$

where, $u(t) \in \mathbb{R}^m \; ; y(t) \in \mathbb{R}^p$

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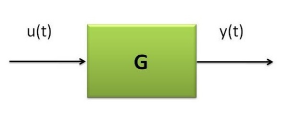

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Special Case

$$u(t) \in \mathbb{R}, \qquad y(t) \in \mathbb{R}$$

Here, G:is a linear map

$$\frac{y(s)}{u(s)} = G(s)$$

$$y(s) = u(s) * G(s)$$

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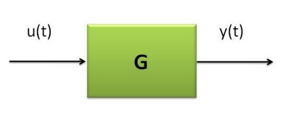

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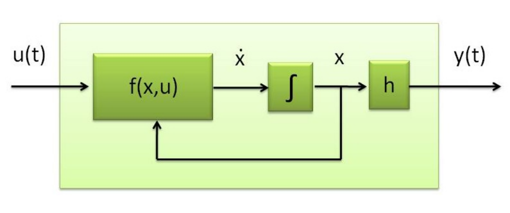
$$\begin{aligned} \frac{y(s)}{u(s)} &= G(s) \\ y(s) &= u(s) * G(s) \\ y(t) &= \int_0^\infty G(t-\tau).u(\tau)d\tau \end{aligned}$$

System description

A dynamical system can be expressed as

$$\dot{x} = f(x, u)$$

then, $y = h(x)$



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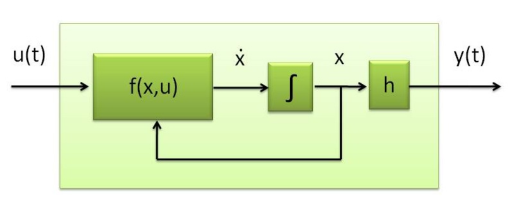

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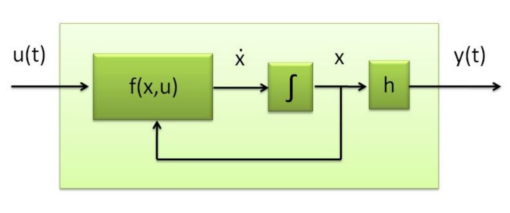

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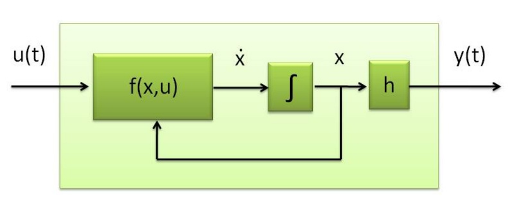

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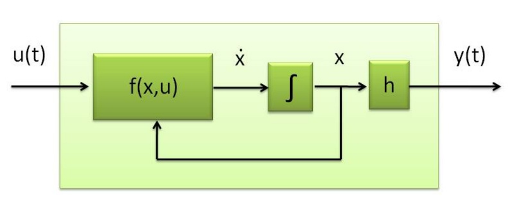

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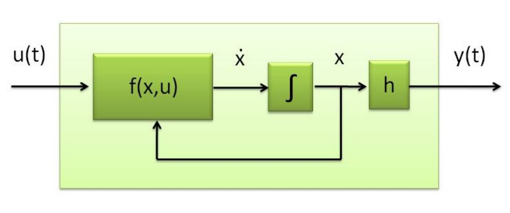

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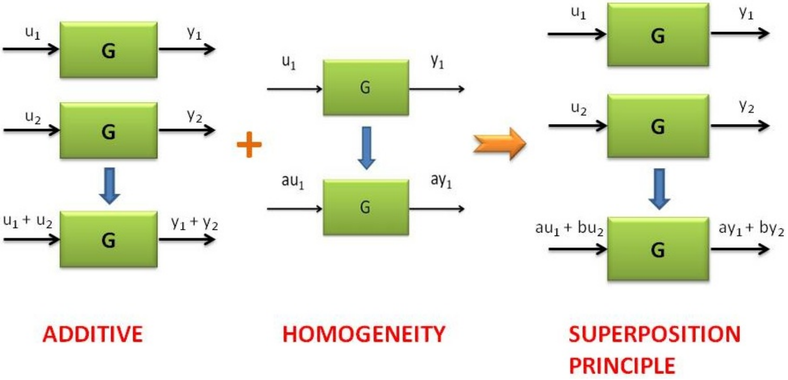
Linear system

Definition

A system is said to be linear in terms of the system input u(t) and the system output y(t) if it satisfies the additive and homogeneity property which implies that it satisfies the superposition principle.



Approximation to linear system

The non linear system equations are

$$\begin{aligned}\dot{x} &= f(x,u)\\ y &= h(x)\end{aligned}$$

is approximated to a linear system of the form

$$\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx\end{aligned}$$

where, $A = \frac{\partial f(x,u)}{\partial x}\Big|_{x_*}$, $B = \frac{\partial f(x,u)}{\partial u}\Big|_{u_*}$, $C = \frac{\partial h(x)}{\partial x}\Big|_{x_*}$

Now,

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Now it is easy to arrive after approximation!

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Introduction to State space

Definition of Space

The state of the system at time $'t_0'$ is the minimum information needed to uniquely specify the system response given the input variable over the time interval $[t_0, \infty]$

State space representation

The state space equations of the system is

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where

$x \in \mathbb{R}^n$; $u \in \mathbb{R}^m$; $y \in \mathbb{R}^p$

$A \in \mathbb{R}^{n\times n}$ $B \in \mathbb{R}^{n\times m}$
$C \in \mathbb{R}^{p\times n}$ $D \in \mathbb{R}^{p\times m}$